**Problem:** if u = x2 – y2 and v =  then show that both **u** and **v**  satisfy the Laplace’s equation but **u+iv**  is not an analytic of z .

**Solve:**

**Given that,**  u = x2 – y2 and v =

So

Now = ()

=

=

= ()

=

=

=

=

= ()

=

=

=

So =

=

=

=

Now (ii) + (iv) gives,

+ = 2-2=0

And (vi) + (viii) gives,

+ =

=

= 0

So u and v both satisfy Laplace’s equation.

From (i) and (vii) we have,

**≠**

And from (iii) and (v) we have,

≠ -

So u + iv is notanalytic function of z , **Showed.**

**Problem** : Show that u(x,y) is harmonic and find the harmonic conjugate v(x,y) of u(x,y) and the corresponding analytic function f(z) = u+iv.

1. u = 2x(1-y)
2. u = ex{x Cos y – y Sin y}

**Solve:**

**1) Give that,**

u=2x(1-2y)=2x-2xy………………….(i)

Now , ux=2-2y and uy=-2x

Uxx=0 uyy=0

**Hence uxx+vxx=0+0=0 , So this equation is harmonic.**

**2nd parts :**

**Let v be harmonic conjugate of u by C-R equation ux = vy and uy = -vx**

**so** vy  = ux =2-2y………………………(ii)

**Integrating (ii) with respect to y and we get,**

V=2y – y2+f(x)…………………………..(iii)

**Differenting (iii) with respect to x and we get,**

Vx=0+f’(x)

=> -uy=f’(x)

=> 2x=f’(x)

**so f(x) = x2+c**

**putting f(x) in (iii) then we get,** v=2y-y2+x2+c

**3rd parts :**

Now f(z) = u+iv

=2x(1-y)+i(2y-y2+x2+c)

=2x-2xy+2iy-iy2+ix2+ic

=2x+2iy+ix²+i²2xy+i³y³+ic

=2(x+iy)+i(x²+2ixy+i²y²)+ic

=i(2+iy)2+2(x+iy)+ic

=iz2+2z+ic. Ans:

**2) Let u = ex{x Cos y – y Sin y}**

=ex.x cos y – ex y sin y ……………(i)

Now, ux = (ex+xex)cos y - ex y sin y

uxx = (ex + ex +xex) cos y – exy sin y

= 2ex cos y + xex cos y – ex y sin y

and uy = - ex x sin y – ex (sin y + y cos y)

uyy = - exx cos y – ex (cos y + cos y – ysin y)

= - ex x cosy – 2ex cos y + exy sin x

**Here uxx + uyy = 0**

**So u is the harmonic.**

**2nd part:** Form C-R equations , we have

ux=vy and uy =-vx

Here ux = ex cos y + xex cos y – ex sin y [ux=vy]

ux =vy =ex cos y + xex cos y – ex sin y…………….(ii)

**Integrating (ii) with respect to y we get,**

V=ex sin y + xex sin y – ex [-y cos y].(-cos y)dy] + f(x)

= ex sin y + xex sin y + ex cos y – ex sin y + f(x)

=> v = xex sin y + exy cos y + f(x)…………………(iii)

**Now differenting (iii) with respect to x, we get**

Vx =xex sin y + ex sin y + ex y cos y +f’(x) [uy=vx]

-uy= xex sin y – ex sin y – ex y cos y

=> -(-ex x sin y – ex sin y – ex y cos y) = xex sin y + ex sin y + ex y cos y + f’(x)

=> f’(x) = 0

**So f(x) = 0+c**

**Putting the value of f(x) in (iii) we get,**

V=xex sin y + exy cos y + c

**3rd part:**

Now f(x) = u+ iv

=ex x cos y – ex y sin y + ix ex sin y + iexy cos y+i

=ex [x(cos y +i sin y) + i2y sin y + iy cos y]+c’

= ex [x(cos y +i sin y) + iy (cos y + i sin y)]+c’

=ex(x+iy)(cos y + i sin y) + c’

= ex.z.eiy+c’

=z.ex+iy+c’

=z.ez + c’  **Ans:**

**Problem: show that the following function are analytic**

1. **f(z) = 3z-iz ii) f(z) = sin z**
2. **f(z) = x2+iy3 iv) f(z) =**

**solve : i)**

**Given that, f(z) = 3z –iz**

=> f(z) = 3(x+iy)-i(x+iy)

=> f(z) = 3x + 3iy – ix –i2y

=> f(z) = 3x + 3iy – ix +y2

=> u+iv = (3x + y) + i(3y – x)

**Equating the real and imaginary parts , we get**

u = 3x +y and v = 3y-x

so = 3 and = -1

= 1 and = 3

**Hence = and =**

**… Thus cauehy riemann equation satisfiy every where. So f(z) is analytic every where.**

**Solve : ii)**

**Given that, f(z) = sin z**

f(z) = sin ( x+iy )

= sin x cos iy + cos x sin iy

=> u + iv = sin x coshy + i cos x sinhy [cos iy = coshy & sin iy = i sinhy]

**Equating the real and imaginary part , we get**

u = sin x coshy and v = cos x sinhy

= and

=

So = and

**Thus C-R equation satisfied every where so f(z) is analytic every where.**

**Solve: iii)**

**Given that,** f(z) = x2 +iy3

u+iv = x2 +iy3

**Equating the real and imaginary parts , we get**

u = x2and v=y3

and

0 and = 3y2

**Here and**

**So the given function does not Cauehy - Riemann ekuation and it is not analytic**

**Solve : iv)**

**Given that,** f(z) =

= =

=> u + iv = = [cos2xy + i sin 2xy]

Now equating the real and imaginary parts , we get

u = and v = e2xy.sin2xy

so =

=

=

=

**So**  = and =

Thus u and v satisfy the Cauchy Riemann equation. So f(z) is analytic every where .

**Problem:**  if f(z) is a function then show that ( + ) |f(z)|2 = 4|f’(z)|2

**Solve:**  iff(z) = u + iv

=> |f(z)|2 = |u+iv|2

= u2 + v2

Now f’(z) = ux + ivx

= vy – iuy **[ using C-R equation ]**

So |f’(z)|2 = ux2 + vx2 = ux2 + yy2

Now |f(z)|2  = (u2 + v2 ) = u2 + v2

=2uux + 2vvx

So |f(z)|2 = 2(uuxx + ux2 + vvxx + vx2 …………………(i)

Similarly |f(z)|2 = 2(uuyy + uy2 + vvyy + vy2 …………………(i)

Adding (i) and (ii) we get,

=> ( + )|f(z)|2 = 2 [ u ( uxx+ uyy) +ux2 + v( vxx + vyy) + vx2 + uy2 +vy2 ]

= 2 [ u.0 + ux2+ vx2 + uy2 +vy2 + v.0 ]

=2[ |f’(z)|2 + |f’(z)|2  ]

4|f’(z)|2  **proved.**